Diffusion

and a Key-Recovery Attack on a WM Scheme by Li and Yuan

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Do not reuse the key

Andrew Ker

- Keys *are* reused in cryptography
  - The one-time pad is not practical
- The solution is diffusion
  - Each key bit is spread widely across output
  - Dependendy between key and output is *too complex* for analysis
- We shall see *lack of diffusion* later (stay awake)

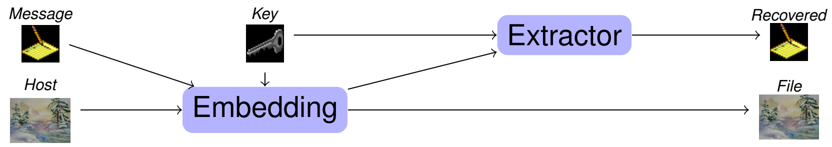
Watermarking is not Cryptography

Ingemar Cox

If we don't study watermarking as a cryptographic problem,
how do you know that cryptanalysis cannot break it?

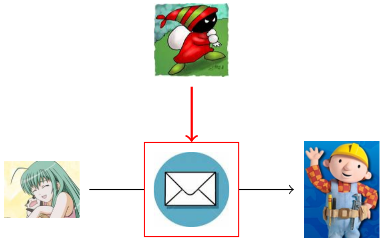
- If it *can be* cast as a cryptographic problem
  - you have to use cryptology in the design,
  - because your adversary may use it in the attack
- Cryptology is a methodology, not just a series of primitives
- Admittedly, Li-Yuan is better seen as a layered system
  - We break the cryptological layer
  - We do not touch the watermarking layer (embedding)
- i.e. Cox' view may stand . . . for now

Digital Watermarking



- Digital Watermarking 'hides' a message in another file (the *host*)
- The watermarked image can replace the cover
  - *Perceptually Equivalent*
- In fragile watermarking
  - The host cannot be modified without destroying the hidden message
- In robust watermarking
  - The hidden message cannot be modified or destroyed without destroying the host

The Authentication Problem



- Alice sends a message to Bob
- Bob wants to assure that it is authentic
- Eve wants to modify the message and fool Bob

Authentication Techniques

- Cryptograhic solutions
  - Message Authentication Code (MAC) – Secret Key
  - Digital Signatures – Public Key
- Watermarking embeds Authentication Information in the file
  - no appended signature to handle
  - everything fits into the host file format
- Creating and attacking the authentication information
  - remains a cryptological prolem
  - layered system (here Cox and I agree)
- It does not matter if the designer agrees
  - I, as an attacker, can use cryptology anyway

Cryptography

Authentication Techniques

- Cryptograhic solutions
  - Message Authentication Code (MAC) – Secret Key
  - Digital Signatures – Public Key
- Certificate of Authenticity (Signature or MAC)
  - ... appended to the message
  - does not fit into standard file formats
- Only Alice can produce a valid certificate
  - well-studied and trusted technology
  - mathematical security

Authentication Watermarking

- Authentication information is embedded in the file
  - no appended signature to handle
  - everything fits into the host file format
- Some watermarking systems offer extra advantages
  - localisation of changes/errors
  - further analysis of modification processes
- Creating and attacking the authentication information
  - remains a cryptological prolem
  - layered system (here Cox and I agree)

The Li-Yuan System

Symbols and definitions

- $M \times N$ 8-bit grayscale image image $\mathcal{I}(x, y)$
- Security parameter b
  - Discard the b least significant bits of each pixel
  - $\rightarrow$ *significant image* $\mathcal{S}(x, y)$
- Secret watermark image **w**
  - $M \times N$ matrix of b-bits per item (pixel)
  - A shorter key can be expanded using a secure PRNG
- Let $a(x, y)$ denote the authentication information
  - b bits per pixel (to be computed)
- The watermarked image will be generated as

$$\mathcal{W}(x, y) = 2^b \mathcal{S}(x, y) + a(x, y),$$

Extraction and Authentication

- Extraction
  - $v(x, y)$ is computed (hash of $\mathcal{S}$)
  - $a(x, y)$ is extracted directly ($= \mathcal{I} \mod 2^b$)
  - Extracted watermark $w'(x, y) = v(x, y) \oplus a(x, y)$
  - Secret watermark $w(x, y)$ is known
- $w'(x, y) \neq w(x, y)$ indicates an error

A non-cryptographic hash

Calculating the authentication information

- Main challenge: calculating $a(x, y)$
  - if Eve can calculate $a(x, y)$ for a false image,
  - ... she has broken the scheme

For each pixel (x, y),

- Consider a $k \times k$ square region $N_k(x, y)$ around it
- A b-bit hash $v(x, y)$ is calculated from
  1. $\mathcal{S}$ on $N_k(x, y)$
  2. least significant bits of **w** on $N_k(x, y)$
- $a(x, y) = v(x, y) \oplus w(x, y)$ replace b LSB-s

The problem

- Each watermarked pixel (x, y) depend on 26 key bits
  - This includes 5×5 bits of $\kappa := w \mod 2$
  - And one extra bit $w(x, y)$ 'encrypting' $v(x, y)$
- A key principle of cryptography is diffusion
  - Each output bit should depend on every key bit
- Dependence on 26 bits is insufficient
  - An exhaustive search is possible
  - work on 25 bits of κ at a time
- Proper *Diffusion* would prevent the attack

Assumptions

- We need two known, watermarked images $\mathbf{x}_1$, $\mathbf{x}_2$
  - One image is not sufficient
  - More images give faster decoding
- We assume $k = 5$
  - We sketch improvements to be feasible for $k > 5$
  - ... but the details remain for future work
  - ... the improvements depend on image properties
- We assume $b = 2$
  - $b > 2$ makes the attack faster
  - $b = 1$ makes it slower, but additional images can compensate
  - (Note that Li and Yuan claim that increasing b increases security)

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The idea

The first round

- Consider a 5×5 block at a time
- Exhaustive search : 2^{25} possible subkeys $\kappa | N_5(x, y)$
- For each tentative subkey $\hat{\kappa}$
  1. Extract watermark $w_i'(x, y)$ $(i = 1, 2)$ from $\mathbf{x}_i$
  2. Compare w_1' and tentative key
     - $w_1'(x, y) \mod 2 \neq \hat{\kappa}(x, y)$: reject $\hat{\kappa}$
  3. Compare w_1' and w_2'
     - $w_1'(x, y) \neq w_2'(x, y)$: reject $\hat{\kappa}$
- Three (3) bit comparisons are made
  - On average, one key in eight (2^3) pass the test

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How to proceed

The rest of the idea

- Each round considers a new 5×5 block
  - ... overlapping with the first
- Number of possible keys *increase at first*
- Rounds 2-3 add five key pixels each
- Round 4 add only 1 ($6 \times 6 = 36$ pixels total)
- Rounds 5 and 7 add five pixels each
- Rounds 6, 8, and 9 add one pixel each
  - $7 \times 7 = 49$ pixels covered after Round 9
- Thereafter: expected number of tentative keys *will decrease*

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Strong cryptography

- Two problems
  - Short key : weak ‘cryptography’ at best
    - ... exploited by the basic attack
  - Insufficient diffusion : non-cryptographic
    - ... exploited by improvements (paper only)
- $a(x, y)$ requires the properties of a MAC
  - Eve knows several watermarked images (with $\mathcal{S}$ and a)
  - Eve cannot produce a new image $\mathcal{S}'$ with matching authentication information (a').
- A proper MAC would prevent our attack
  - There are some works using MAC-s in authentication watermarking
  - ... and some works recognise the importance of cryptography, but use the wrong cryptographic properties.

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The Design Parameters

- Decreasing b
  - Fewer keys are excluded in each round
  - But hash collisions become more frequent
- Increasing k
  - More keys to consider per round
  - However, if a monochrome region can be found in the image,
    - Only k^2 (not 2^{k^2}) keys have to be considered
    - By exploiting the simple additive structure of $S(x, y)$
    - And increasing k will have marginal effect...

Conclusion

- Key-Recovery Attack Algorithm on Li and Yuan's Scheme
- Cryptological principles apply
  - If the designer ignores them,
  - then the attacker can exploit them
- Open problem
  - Implement and test the algorithm
  - How secure are other watermarking systems?