Learning mathematics through classroom interaction

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Abstract

We have used flipped classroom for three years running in our module on *Discrete Mathematics* for the undergraduate programme in computer engineering. Online video lectures free up classroom time for active learning. The success of flipped classroom depends on what replaces the lectures. Debating exercises and solutions is generally more rewarding than individual problem solving. Even though mathematics is not usually seen as a 'chatty' subject, there is a strong theoretical basis to increase all forms of social interaction also in mathematics education, from informal chatting to more formal discussion. Feedback is a well-known bottleneck when students work individually with exercises. Through dialogue, the participants can get prompt feedback both from peers and from tutors. Discussion may also help to develop the necessary vocabulary and mathematical language. In this paper we report on a detailed survey conducted in the 2015 class. We will give an overview of different learning activities used, and evaluate them in terms of the student survey and their theoretical justification.

Introduction

Flipped classroom is a hot topic at all levels of education. Modern consumer-end computers have made anybody a potential film and video producer, and many teachers and lecturers take advantage of the new technology to move their lectures from the classroom to the web. Thus class time is free to be used for other activities. "In a flipped classroom, the information-transmission component of a traditional face-to-face lecture (…) is moved out of class time. In its place are active, collaborative tasks" (Abeysekera and Dawson, 2014). In other words, *active learning* is an inseparable part of the flipped classroom. Thus there are as many approaches to flipped classroom as there are to active learning. Finding the optimal approach for a given subject, in a given culture, for a particular class of students, is non-trivial. At NTNU Ålesund we have used flipped classroom in the module on *Discrete Mathematics* in the third semester of the computer engineering course. In this paper we will discuss a variety of learning activities used in this module over three years. The analysis is based on a quantitative survey carried out with the 2015 cohort. A qualitative report has been made previously (Schaathun, 2015).

Literature review

Higher education is dominated by *teaching by telling*, or *the 'transmission method' of teaching* (Sotto, 2007). During a lecture, information is *transmitted* from the teacher to the student. Exercises are often used in addition to lectures, but mostly for rehearsal which assumes that the students have already learnt the material from transmission. Many voices are speaking up against the traditional lecture (e.g. Mazur, 2009). The alternative is *active learning*, which includes any instructional method that engages students in the learning process (Prince, 2004). The most extreme forms of active learning, *learning by doing*, are dominated by self-directed discovery on the part of the students, popular under different names (e.g. discovery learning, problem-based learning). Yet there is no empirical evidence to support it (Mayer, 2004). It is however important to observe that active learning also includes less obvious activities like *teaching by questioning* (Mazur, 2009) or inclusive discussions in groups or in class.

Active learning is motivated by the constructivist position that knowledge is constructed by each learner. It is well known in cognitive psychology (Anderson, 2015) that deep cognitive processing is necessary to commit new information to long-term memory. This is often modelled as organising the information into schemata. Mayer (2004) speaks of the *constructivist fallacy*, where the popular interpretation of constructivism fails to distinguish between cognitive activity and behavioural activity. The former is essential for learning, while the latter is neither sufficient nor necessary.

Cognitive Load Theory can explain why overdoing active learning fails (Clark et al., 2005). New information has to be processed in working memory before it can be committed to long-term memory, and working memory has very limited capacity (Anderson, 2015). Complex problems requiring complex information to solve, can only be managed when the necessary information has been encoded in a schema. Many authors have argued for small manageable exercises which the students can solve without excessive cognitive load (e.g. Colburn, 1822 and Sotto, 2007). Abeysekera and Dawson (2014) argue that the flipped classroom helps students manage cognitive load, because they can regulate the pace of the videos watched.

The module design

The module is worth 10 ECTS credits and run for fourteen weeks including two weeks of exam revision sessions, using

- **Classroom sessions:** Two-hour session three times a week.
- **Video:** Lectures are given on video. Typical length is 3–7 minutes long for videos created 2015, while older ones tended to be longer, up to about 15 minutes.
- **Web pages:** Simple static web pages make all teaching material available, including exercises, reading lists, video, et cetera.

Two alternative textbooks were suggested, but neither gives a complete coverage. The syllabus was defined by the videos and the exercises, with emphasis on the latter.

The assessment is a single written exam at the end of term. However, to be allowed to sit the exam, the student has to complete a compulsory assignment. Such assignments are common in similar modules in Ålesund. The primary intention is to force students to work steadily throughout the semester. The compulsory assignments follow a format suggested by Kristina Edström in her keynote at *MNT-Konferansen* in Bergen March 2015. One of the three weekly class room sessions is used for a student-led tutorial. Typically, six problems are assigned, and the students have to prepare to *present* solutions to as many as possible. At the start of the session every student tick which solutions they can present on a class list. For each problem one student is drawn randomly to present. Each student is required to have at least 40% ticks over the semester to sit the exam. If a student is caught bluffing, being drawn to present something he is clearly not prepared for, all ticks that day are cancelled.

The other two classroom sessions are used in a flexible way, allowing for improvisation. The starting point is always a set of exercises. Originally, in 2013 the default was individual seat work, although the students were free to collaborate if they pleased. The module convener was available to answer questions. Over the years, we have made a gradual shift towards more group work and plenary discussion. In the 2015 delivery, the session would always start with a plenary discussion either about new material in the videos or about the first problems assigned. Discussing the solution of an exercise in full class would aim to use student input as far as possible, step by step through the problem. Thus an idea to approach or a single step could be heard and praised. Students who cannot see the full solution, can still get positive feedback and see how *their* idea can be brought forward to a complete solution. When several solutions are possible, each one can be heard.

At any time, when appropriate for the exercise or step at hand, the class can switch to group or individual work. Individual work is suitable for computations which the students should know but need more practice to automatise. Group work is very suitable to discuss different solution alternatives. If the class is stuck on a piece of theory, a mini-lecture can be inserted to fill the gap. It is important to note that the activities in the classroom are decided on the spot, based on gut feeling and a good rapport with the students.

Evaluation

The data were gathered in the form of paper questionnaires which were handed out and completed in one of the last student-led tutorials. Because of unusually poor attendance this week it was repeated a couple more sessions, including the following student-led tutorial. We received 26 completed questionnaires. A total of 32 students sat the exam. A few students occasionally attended class and may have completed the questionnaire without sitting the exam. The questionnaire was anonymous, without any question which could reveal identities.

<table>
<tr><td colspan="2" rowspan="2"></td><td colspan="5">1. How do you think flipped classroom works?</td></tr>
<tr><td>Very well</td><td>Well</td><td>Neither</td><td>Badly</td><td>Very badly</td></tr>
<tr><td rowspan="4">2. How much do you learn from flipped classroom compared to traditional methods?</td><td>More</td><td>1</td><td>7</td><td>1</td><td>-</td><td>-</td></tr>
<tr><td>Same</td><td>-</td><td>5</td><td>2</td><td>-</td><td>-</td></tr>
<tr><td>Less</td><td>-</td><td>1</td><td>2</td><td>3</td><td>3</td></tr>
<tr><td>Don't know</td><td>-</td><td>1</td><td>-</td><td>-</td><td>-</td></tr>
</table>

Table 1: Overall attitude to flipped classroom.

The overall attitude to flipped classroom is shown in Table 1, based on two slightly different questions. A broad and generic definition of flipped classroom was given in the questionnaire, but the particular activities in *Discrete Mathematics* must be expected to influence their understanding of the concept. With more than half the respondents in favour of flipped classroom, we have a positive bias, but there is a very significant minority who are not well served by flipped classroom in the current version. Six students (23%) think that flipped classroom works 'badly' or 'very badly' (in the following referred to as the negative group), and nine students (one third) think they learn less.

Table 2 gives the student view on the amount of each activity in the classroom. We observe that both plenary and group discussions are well received, and this is interesting because we have used such discussions much more than what is common in higher education mathematics. Other questions confirm the positive attitude to discussions. More students are positive to group discussions (16/26) than plenary discussions (9/26).

	Too much	Appropriate	Too little	Don't know
Presentation of theory	-	15 (2)	10 (3)	1 (1)
Plenary discussions	5 (3)	21 (3)	-	-
Group discussions	6 (3)	17 (1)	3 (2)	-
Individual exercises	2 (1)	14 (2)	9 (2)	1 (1)
Student presentations	11 (4)	13 (1)	-	2 (1)

Table 2: How do you rate the amount of each activity in the classroom?
Numbers in parenthesis include only students negative to flipped classroom.

When we look at the two most traditional learning activities, namely theory presentations and individual exercises, we find the students split almost in half. The majority think the amount of these activities is appropriate, but about 40% think we have too little.

We also asked the students how much they learn from each activity. To be able to judge the answers on a one-dimensional scale, we assign a score from 0 to 3 to the answers nothing, something, much, and very much, excluding «Don't know». Table Table 3 gives the average score per student. The global average score is $\bar{x} = 1.459$ and the standard deviation is $\hat{\sigma} = 0.15$. Two activities stand out. Reading the textbook is perceived as significantly less effective than any other activity. The teacher presenting solutions on the blackboard is similarly significantly more effective than anything else. The overall impression is that most students learn something from every activity, and few learn very much from anything. Fourteen students did not tick any activity where they learn a lot.

	Very much	Much	Something	Nothing	Don't know	Score
Theory on blackboard	3 (0)	12 (3)	9 (3)	1 (0)	-	1.68
Theory on video	3 (0)	8 (1)	12 (4)	2 (1)	-	1.48
Plenary discussions	1 (0)	8 (1)	15 (4)	1 (1)	-	1.36
Group discussions	2 (0)	12 (2)	11 (4)	-	-	1.64
Individual exercises	4 (1)	10 (1)	11 (4)	-	-	1.72
Own presentation	-	6 (1)	14 (1)	5 (4)	-	1.04
Solutions by students	1 (0)	4 (1)	16 (3)	4 (2)	-	1.08
Solutions on video	3 (1)	12 (2)	8 (2)	2 (1)	-	1.64
Solutions by teacher	8 (3)	9 (2)	8 (1)	-	-	2.00
Read textbook	1 (1)	2 (0)	13 (1)	5 (2)	4 (1)	0.72

Table 3: How much do you learn from each activity?
Numbers in parenthesis include only students negative to flipped classroom.

The most controversial activity in our scheme is the student presentations (compulsory assignments). Table 2 splits the class in half, where one half thinks there are too many presentations while the other thinks the amount is appropriate. In Table 3, the student presentations have scores more than two standard deviations below average. However, this negative attitude to student presentations is not as clear in other questions we have asked. The negative group is consistently less favourable to student presentations. Four of the six negative students think there are too many student presentations, and all six agree that they learn more from written assignments than from blackboard presentations (four strong agree and two weak). The non-negative students are almost balanced on the same question.

There are many plausible reasons why students dislike the student presentations. A majority of students think it is difficult to speak in front of the full class, and it is also important for them to choose when to do compulsory assignments. On these two questions, the negative students do not differ significantly from the rest.

Discussion

These empirical results do not endorse a pure implementation of flipped classroom. Rather, they point to a range of techniques and learning activities which can contribute to an effective module delivery. The results are consistent with observations made by others. In particular, Cognitive Load Theory (CLT) gives a good foundation to understand the results. The traditional 2h lecture gives the students a lot of information with little time to process it. Without time to build schemata, most of the transmitted information is lost. Thus, from a CLT perspective, the problem is not the transmission mode itself, but the duration of each transmission session. The flipped classroom does not solve this problem, unless the students, on their own initiative, put in the time and effort to process new information between videos and before they come to class.

In *Discrete Mathematics* we now interleave transmission teaching and active learning in the classroom, and thus we have moved beyond the pure flipped classroom with which we started in 2013. An overwhelming majority of students, consistently over several overlapping questions, indicate that this transmission teaching is very important for learning. The interleaving achieves two things. It breaks up transmission teaching which would otherwise overload working memory, and gives essential scaffolding during active learning. This is also supported by a study of a module in *Microcontrollers* on the same degree programme (Schaathun and Schaathun, 2016). While a large minority of students want more transmission teaching in the classroom, no learning activity is a popular candidate for reduction.

Both group and class discussions are used in class. There are many reasons why we think this is important. The plenary discussions form important feedback to the teacher. The in-class mini lectures are improvised in response to questions from students. Sometimes it allows the selection of examples best aligned with the previous knowledge. For instance, introducing relations between sets, when we saw that the students had done relational databases in another module the previous week, we focused on this as an example instead of the example which had been prepared, based on object-oriented programming. It is a welcome observation that most of the students approve of the time spent on discussions. We do not have data to analyse why the students are happy with the discussions, whether they enjoy discussing in itself or if they see how the discussion supports and enables other activities.

Theoretical foundation for the use of discussion is found in Vygotsky's work and social constructivist theory. Most learning occurs through interaction, and social interaction is a form of scaffolding. When students present a solution verbally, it is possible to give instant feedback. Instead of individually completing a false solution, the class can stop at the first mistake and discuss why it is wrong and how to get it right. Plenary discussions have the advantage of involving the more knowledgeable teacher, while peer discussions allow more students to take an active role. Foldnes (2016) has also shown that group work is much more effective than individual seat in class. Discussions can also develop the verbal language to discuss the subject matter. It is known (Holm, 2012) that many people who struggle with mathematics lack the language to explain their problems and ask the right question. In a famous experiment, Vygotsky has shown that children talk constantly during problem solving, and preventing speech also prevents the solution. Similarly, inner speech is widely recognised as a key tool in mathematics, and the development of inner speech goes through outer speech (Holm, 2012).

It is not seen in the quantitative data, but in the student-led tutorials we have observed that the development of language leaves a lot to be desired. Even comparably highly skilled students would tend to copy their notes onto the blackboard and fail to explain what they do verbally. This is disconcerting, since it will be difficult for these students to use their knowledge on a professionally applied problem where they need to collaborate with others. Maybe this calls for developing the discussions further with the express goal of training verbal maths skills.

An instructional technique which we have only started to explore is worked examples (Clark et al., 2005), which is essentially a problem with a detailed step by step solution. Worked examples can be used in two different ways. In the inductive approach (Colburn, 1822; Hendrix, 1961) examples are given as the first step of the instruction cycle. General rules and principles are then induced from the examples. In the more traditional, deductive approach, rules and principles are presented first, and examples are introduced later as special cases deduced from the rules. Hendrix (1961) considers the inductive method to be one type of learning by discovery. Worked examples and sample solutions emerge as the one activity which gives the most learning in our study. Qualitative feedback in class also confirms that the students want more solutions. However, most of the examples are used in a deductive fashion; only in 2015, we started to experiment with an inductive approach. Thus while the worked examples are confirmed to be important, we do not have data to compare inductive and deductive use thereof.

Both CLT and the inductive method motivate reconsideration of the use of solutions. Naïvely one might think that solutions are required primarily for difficult problems which few students can solve on their own. Maybe what we really need is a large number of worked examples based on simple problems which everybody understands, to allow students to build up schemata before they try to solve difficult problems.

Conclusion

Teaching is a design problem which cannot be solved by a simple recipe. It is an art form which depends on a close understanding of both the learners and of the subject matter. Cognitive psychology and educational research provides very useful background knowledge to guide the design, but it all has to be interpreted in the concrete context of a specific subject and real students. This paper should mainly be read as an example of *one* approach to delivering a higher education module in mathematics. It is an approach which depends on a range of constituent techniques, and student responses indicate that every technique contribute to the learning. We have not yet found the optimal mix of these techniques, nor do we expect a universally optimal mix to exist. However, we do believe that just lectures and individual exercises make too small a toolbox for a successful maths teacher. We hope that this paper can motivate other teachers to expand their educational toolbox, with new tools and activities from here or elsewhere. We are not writing this paper because our three years of experimentation in *Discrete Mathematics* has given any final answers. Rather we hope to persuade the reader that it is worth taking some years to experiment with new techniques.

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