Using the *t*-distribution

An Estimation Exercise

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13th February 2014

Problem

Exercise

A class of 10 children compete in a 60m race. You record the following times:

$$9.2s, 9.9s, 10.2s, 10.7s, 11.1s, 11.1s, 11.4, 11.5, 11.8, 13.5s$$

Calculate a 95% confidence interval for the mean time.

HØGSKOLEN I ÅLESUND
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Step 1: Sample Mean

The point estimator

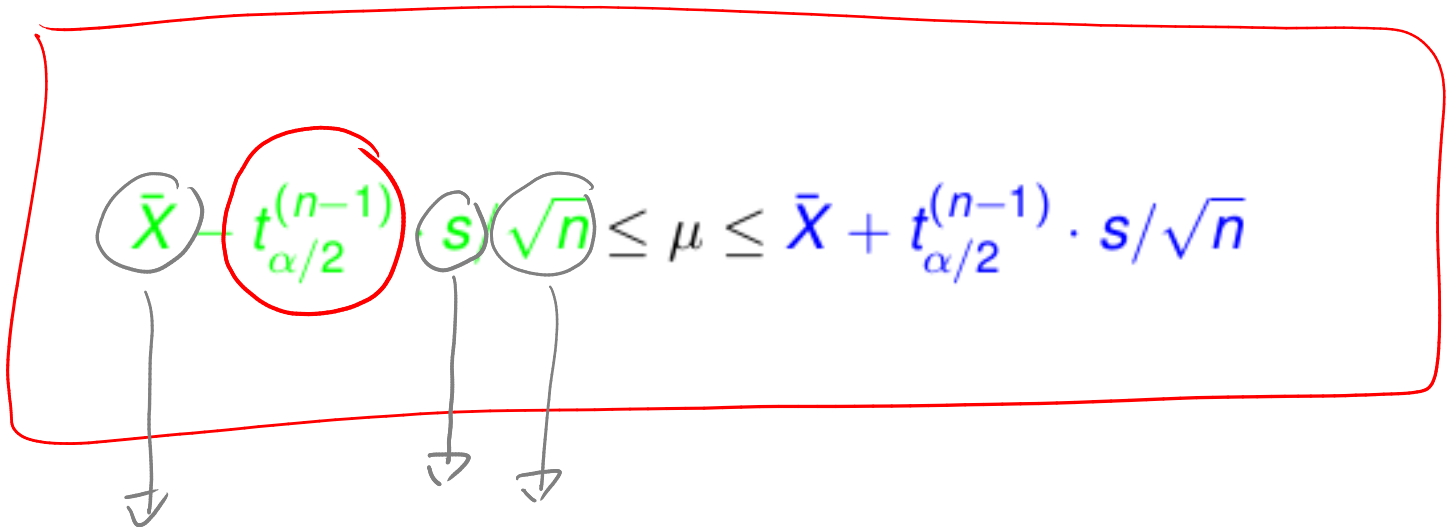
9.2s
9.9s
10.2s
10.7s
11.1s
11.1s
11.4s
11.5s
11.8s
13.5s

Step 2: Sample Standard Deviation

	-11.04s	squared
9.2s	-1.84s	3.3856
9.9s	-1.14s	1.2996
10.2s	-0.84s	0.7056
10.7s	-0.34s	0.1156
11.1s	0.06s	0.0036
11.1s	0.06s	0.0036
11.4s	0.36s	0.1296
11.5s	0.46s	0.2116
11.8s	0.76s	0.5776
13.5s	2.46s	6.0516
110.4s		12.4840

- $\bar{x} = 11.04$
- $s^2 = \frac{\sum(x_i - \bar{x})^2}{n-1} = \frac{12.484}{9}$
- $s^2 = 1.3871$
- $s = \sqrt{1.3871} = 1.1778$

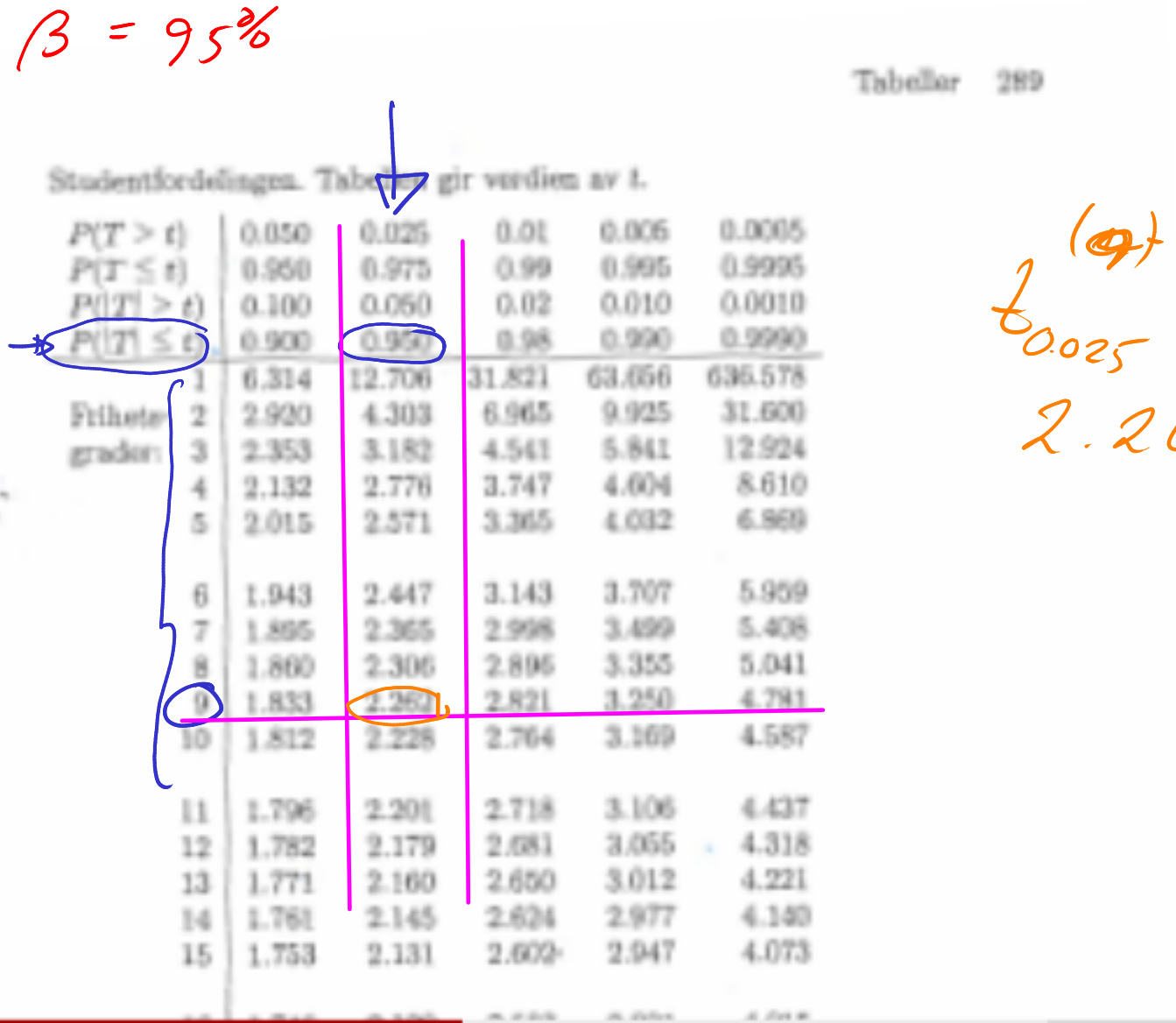
The formula


$\bar{X} - t_{\alpha/2}^{(n-1)} \cdot s/\sqrt{n} \leq \mu \leq \bar{X} + t_{\alpha/2}^{(n-1)} \cdot s/\sqrt{n}$

Step 2: Using a probability table

From Frisvold and Moe

$\beta = 95\%$



Tabeller 289

Studentfordelingen. Tabellen gir verdien av t.

$P(T > t)$	0.050	0.025	0.01	0.005	0.0005
$P(T \leq t)$	0.950	0.975	0.99	0.995	0.9995
$P(\lvert T\rvert > t)$	0.100	0.050	0.02	0.010	0.0010
$P(\lvert T\rvert \leq t)$	0.900	0.950	0.98	0.990	0.9990
1	6.314	12.706	31.821	63.656	636.578
Frihets- 2	2.920	4.303	6.965	9.925	31.600
grader: 3	2.353	3.182	4.541	5.841	12.924
4	2.132	2.776	3.747	4.604	8.610
5	2.015	2.571	3.365	4.032	6.869
6	1.943	2.447	3.143	3.707	5.959
7	1.895	2.365	2.998	3.499	5.408
8	1.860	2.306	2.896	3.355	5.041
9	1.833	2.262	2.821	3.250	4.781
10	1.812	2.228	2.764	3.169	4.587
11	1.796	2.201	2.718	3.106	4.437
12	1.782	2.179	2.681	3.055	4.318
13	1.771	2.160	2.650	3.012	4.221
14	1.761	2.145	2.624	2.977	4.140
15	1.753	2.131	2.602	2.947	4.073

$t_{0.025}^{(9)} = 2.262$

Completing the Solution

$10.2 \leq \mu \leq 11.88$

$11.04s \pm 2.262 \cdot 0.3724$

$$\bar{X} - t_{\alpha/2}^{(n-1)} \cdot s/\sqrt{n} \leq \mu \leq \bar{X} + t_{\alpha/2}^{(n-1)} \cdot s/\sqrt{n}$$

- $\bar{x} = 11.04s$
- $s = 1.1778$
- $n = 10$, $\sqrt{n} = 3.1623$
- $s/\sqrt{n} = 0.3724$
- $t = 2.262$
- $t \cdot s/\sqrt{n} = 0.8425$

$0.3724 \cdot 2.262$

7448

22344

7448

7448

$0.8423688 \approx 0.84$