Calculating a Confidence Interval

First Example

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Problem

Exercise

You are interested in the average height of 6-year olds. In a class of 16 children you measure the following heights in centimeters:

$$109, 114, 115, 118, 119, 120, 121, 121,$$
$$121, 122, 124, 124, 127, 128, 128, 131$$

Suppose you know that the standard deviation is $\sigma = 4$. Calculate a 95% confidence interval for the mean height.

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Step 1: Sample Mean

The point estimator

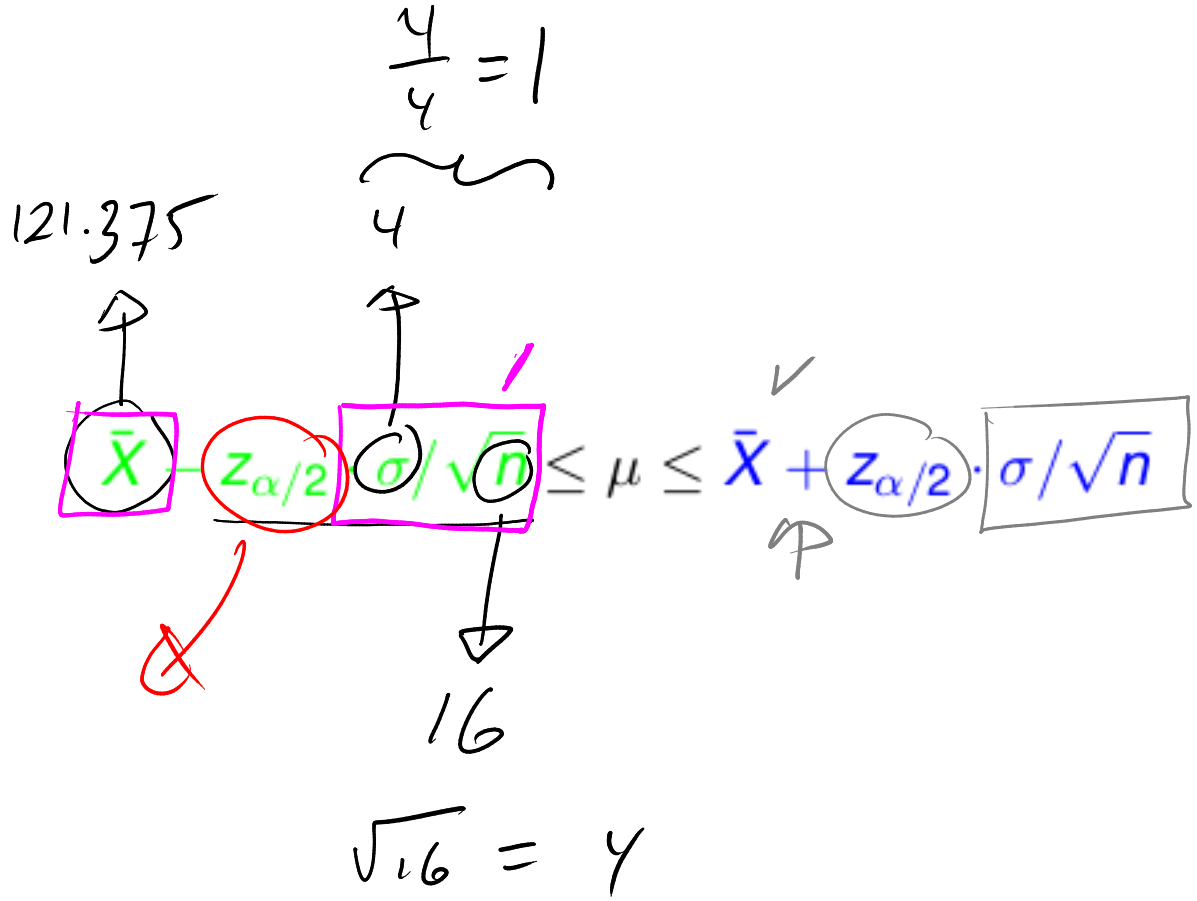
$$\bar{x} = 1942/16 = 121.375$$

109, 114, 115, 118,
119, 120, 121, 121,
121, 122, 124, 124,
127, 128, 128, 131

476	484	488	494

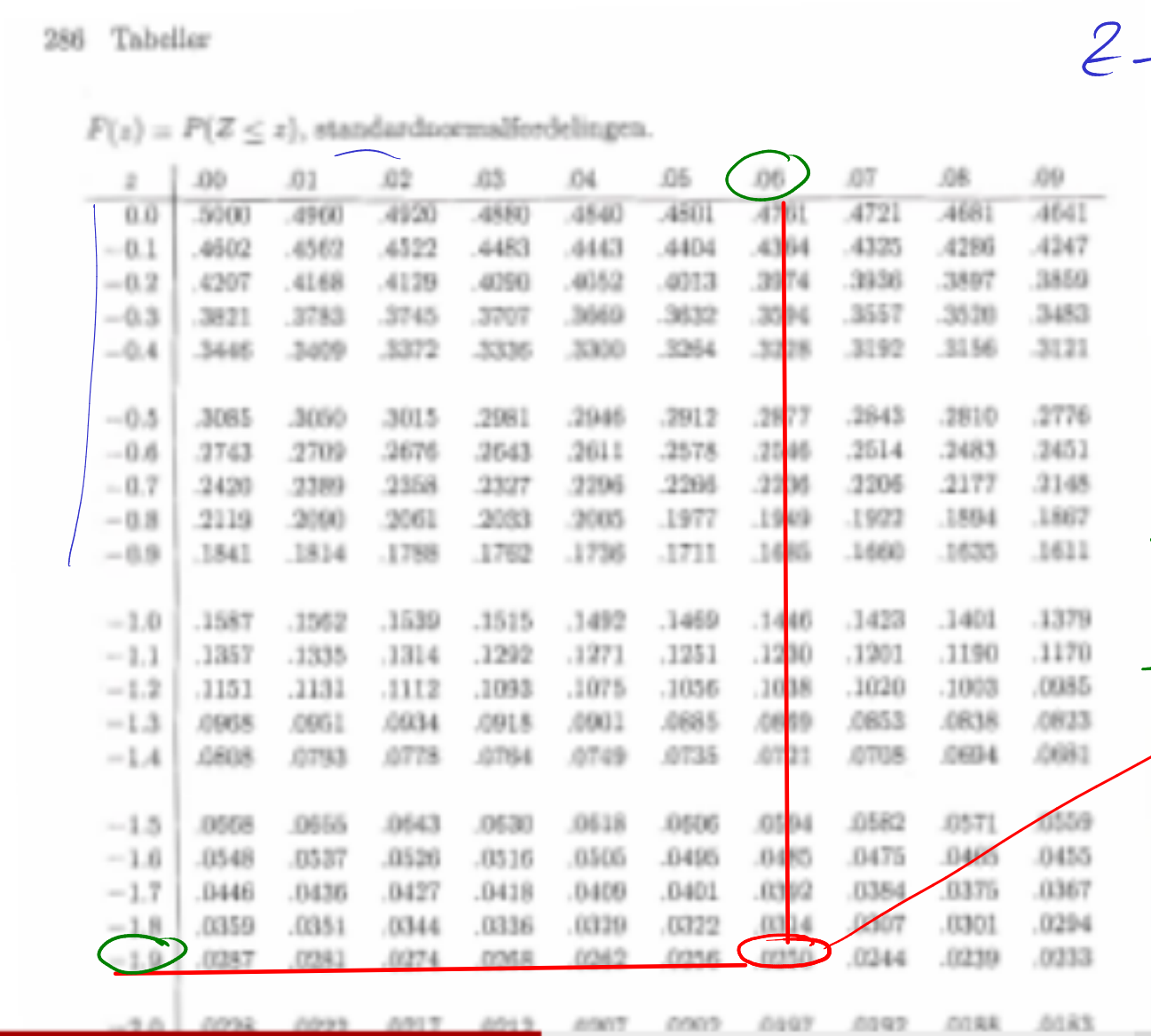
$$476 + 484 + 488 + 494 = 1942$$

The formula


$\frac{4}{4} = 1$
4
121.375
$\bar{X} - z_{\alpha/2} \cdot \sigma/\sqrt{n} \leq \mu \leq \bar{X} + z_{\alpha/2} \cdot \sigma/\sqrt{n}$
16
$\sqrt{16} = 4$

Step 2: Using a probability table

From Frisvold and Moe



286 Tabeller

$F(z) = P(Z \leq z)$, standardnormalfordelingen.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1201	.1190	.1170
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233

z-table

$\beta = 95\%$

$\alpha = 1 - \beta = 5\%$

$\alpha/2 = 0.025$

-1.9

$.06$

$1.96 = z_{0.025}$

Completing the Solution

$$121.375 - 1.96 \cdot 1 \leq \mu \leq 121.375 + 1.96 \cdot 1$$

$$\bar{X} - z_{\alpha/2} \cdot \sigma/\sqrt{n} \leq \mu \leq \bar{X} + z_{\alpha/2} \cdot \sigma/\sqrt{n}$$

$$\mu \in \{119.415, 123.335\}$$

- $\bar{x} = 121.375$
- $z = 1.96$
- $n = 16$, $\sqrt{n} = 4$
- $\sigma/\sqrt{n} = 1$
- $z_{0.0.25} \cdot \sigma/\sqrt{n} = 1.96$

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