The Binomial Distribution

Error Words on the BSC

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Words on the BSC

- Send an n-bit word $\mathbf{x}$ over BSC(p)
- The received word is $\mathbf{R} = \mathbf{x} \oplus \mathbf{E}$
  - where $\mathbf{E}$ is a random error vector

Problem

Let $T = w(\mathbf{E})$ be the number of bit errors.
Describe the probability distribution of T.

We will solve the problem in two steps.

Distribution of the error vector

- The error word $\mathbf{E}$ is a stochastic variable.
- We can start with the probability distribution of $\mathbf{E}$.

Exercise

What is the probability that $\mathbf{E} = (0100110)$*?*

Solution

Distribution of the error vector

$P(\mathbf{E} = (0100110)) =$

$(1-p) \cdot p \cdot (1-p) \cdot (1-p) \cdot p \cdot p \cdot (1-p) = p^3 \cdot (1-p)^4$

$w(\bar{e})$

$P(\bar{E} = \bar{e}) = p^{w(\bar{e})} \cdot (1-p)^{n-w(\bar{e})}$

Probability of an error word

The probability of a given error word **e** *depends only on the number of bit errors* $w(\mathbf{e})$.

$$P(\mathbf{E} = \mathbf{e}) = p^t(1-p)^{n-t}, \quad (1)$$

$$\text{where } t = w(\mathbf{e}). \quad (2)$$

n is the length of $\bar{e}$

Counting possible error words

The probability of a given error word depends only on the number of bit errors.

Exercise

How many n-bit words exist with Hamming weight t?

This is a fundamental counting problem.

Solution

Counting possible error words

- How many *n*-bit (error) words exist with Hamming weight *t*?
- Choose *t* error positions out of *n* possible.
- How?

This is what the binomial coefficient is for...

$$\binom{n}{t} = \frac{n!}{t!(n-t)!}. \qquad (3)$$

Solution

Counting possible error words

- How many n-bit (error) words exist with Hamming weight t?
- Choose t error positions out of n possible.
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This is what the binomial coefficient is for...

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The probability of t errors

What is the probability $P(T = t)$?

- Multiply
  1. the probability of a given t-error word
  2. the number of possible t-error words

$$P(T = t) = \binom{n}{t} p^t (1 - p)^{n-t}$$

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Closure

- Let X be the number of successes in n Bernoulli trials with success probability p
- X is binomially distributed with probability p
- We write $X \sim B(n, p)$

- A bit transmission on BSC is a Bernoulli trial
- The number X of bit errors on an n-bit word
  - is binomially distributed
- We write $X \sim B(n, p)$

Exercise

What other examples of binomially distributed variables can you find? Review binomial distributions in the textbook (Frisvold and Moe).