GENETIC ALGORITHMS: THE BINARY GA

Date: Friday 1 April 2016

Course: Functional Programming and Intelligent Algorithms

Lecturer: Robin T. Bye

Components of binary GA

Algorithm flow

1. Define cost function, cost, variables. Select GA parameters.
2. Generate initial population.
3. Decode chromosomes.
4. Find cost for each chromosome.
5. Select mates for reproduction.
6. Mating.

Algorithm flow

7. Mutation.
8. Check stopping criteria
   - IF (reached max number of iterations OR converged) THEN stop
   - ELSE go to Step 3.

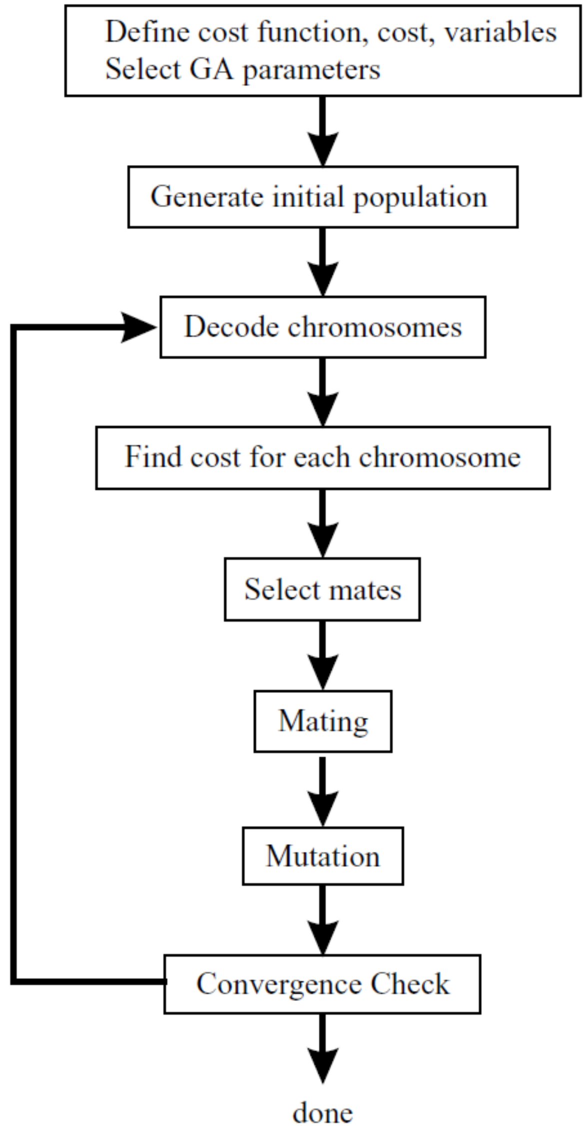


Figure 2.2 Flowchart of a binary GA. Adapted from [1].

Variables and cost function

- Nvar-dimensional problem → chromosome has Nvar variables (genes), i=1,.., Nvar
- chrom = [p1,p2,...,pNvar]
- Cost = f(chrom) = f(p1,p2,...,pNvar)
- Example: 2D height map in xy-plane
  - chrom = [x,y]
  - cost = height = f(chrom) = f(x,y)

Variables and cost function

- If too many variables → slow GA
- Eg. $f = 2x+3y+z/10000+\sqrt{w}/9876$ with constraints $1 \le x,y,z,w \le 10$
- Due to constraints, z, w terms relatively small → ignore: $f = 2x + 3y$
- Variable interaction (epistasis)
  - GA good for medium/high interaction
  - Random search good for high interaction
  - Minimum-seeking good for low interaction

Encoding/decoding

- Encoding: Convert variable values to binary genes
- Decoding: Convert binary genes back to human-readable variable values
- Example:

Bin	Dec	Numbers	Alt. Numbers	Colour	Speed
00	0	10	13.75	Red	Slow
01	1	20	21.25	Green	Medium
10	2	30	28.75	Blue	Fast
11	3	40	36.25	Yellow	Superfast

Encoding/decoding

- Example continued:
  - gene1 = 01 ⇔ medium
  - gene2 = 10 ⇔ fast
  - gene3 = 11 ⇔ superfast
  - gene4 = 00 ⇔ slow
- chrom = [gene1, gene2, gene3, gene4] =[01101100] =[med,fast,supfast,slow]

Encoding/decoding

- Goal: Sort categories in increasing order (slow,medium,fast,superfast)
- Cost: 0 for correct place, 1 for one place off, 2 for two places off, etc.
  - [01101100] =[medium,fast,superfast,slow]
    - $\rightarrow$ Cost = 1 + 1 + 1 + 3 = 6
  - [00100111] =[slow,fast,medium,superfast]
    - $\rightarrow$ Cost = 0 + 1 + 1 + 0 = 2

Encoding/decoding

- Number of bits Nbits in chromosome:
  - Ngene = number of bits in each gene/var
  - Nvar = number of genes/variables
  - Nbits= Ngene × Nvar= number of bits

Population

- Set of Npop chromosomes
- Each chromosome has Nbits
- Represented as matrix of binary digits
- Dimensions are Npop × Nbits
- Initial population randomly assigned:
  - pop=round(rand(Npop, Nbits));

Natural selection

1. Rank chromosomes (low cost better)
2. Only keep best fraction (selection rate Xrate) of Npop chromosomes → Nkeep = Xrate × Npop chromosomes survives
3. Let kept chromosomes mate and replace discarded chromosomes

Pairing methods

- From top to bottom (1+2, 3+4, etc.)
- Uniform random pairing
- Weighted random pairing
  - rank weighting
  - cost weighting
- Tournament selection
- Others

Mating

- Randomly pick a crossover point
- Parent1 passes left-bits to offspring1 and right-bits to offspring2
- Parent2 passes left-bits to offspring 2 and right-bits to offspring1
- p1 = [L1 | R1], p2 = [L2 | R2] →
- o1 = [L1 | R2], o2 = [L2 | R1]
- Other schemes exist

Elitism

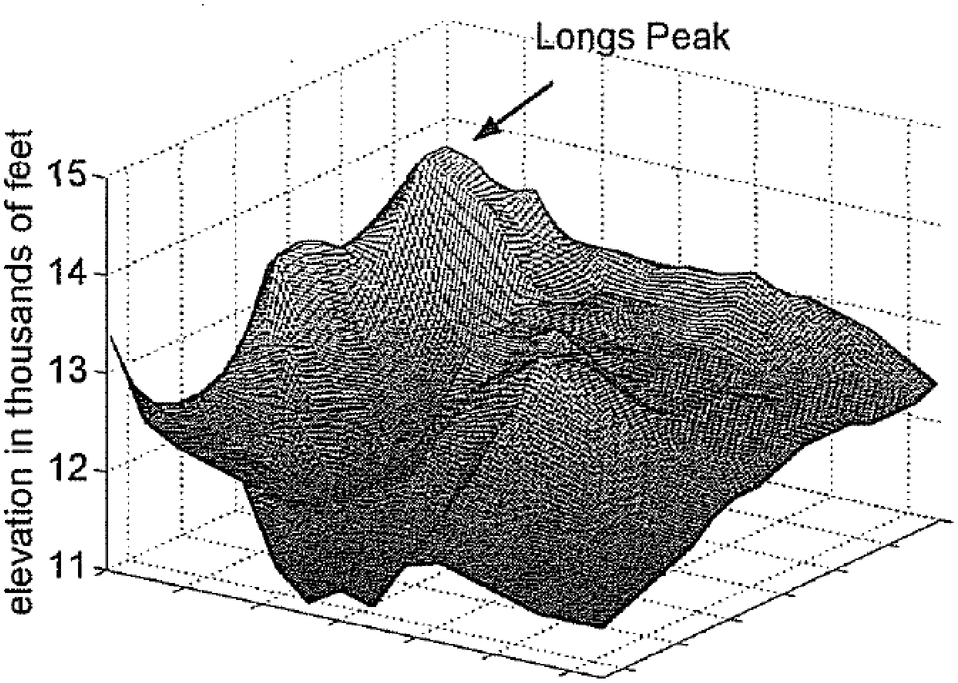
- Always keep best chromosome in population and never mutate it!
- Do not throw away a good solution!

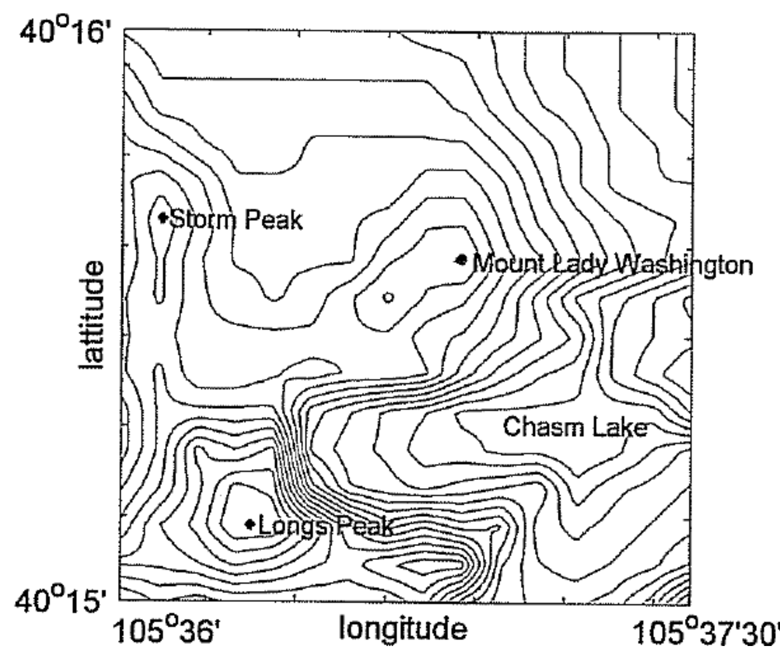
Next generation

- Insert offspring into population
- Recalculate costs and repeat process until
  - convergence
  - max number of iterations reached
  - you are happy for some reason

Example 2D problem

Example 2D problem





Three-dimensional view of the cost surface with a view of Long's Peak.

Adapted from [1].

Example 2D problem

Encoding

TABLE 2.2 Binary Representations

Variable	Binary	Decimal	Value
Latitude	0000000	1	40°15′
Latitude	1111111	128	40°16′
Longitude	0000000	1	105°36′
Longitude	1111111	128	105°37′30″

$$chromosome = \left[\underbrace{1100011}_{x}\underbrace{0011001}_{y} \right]$$

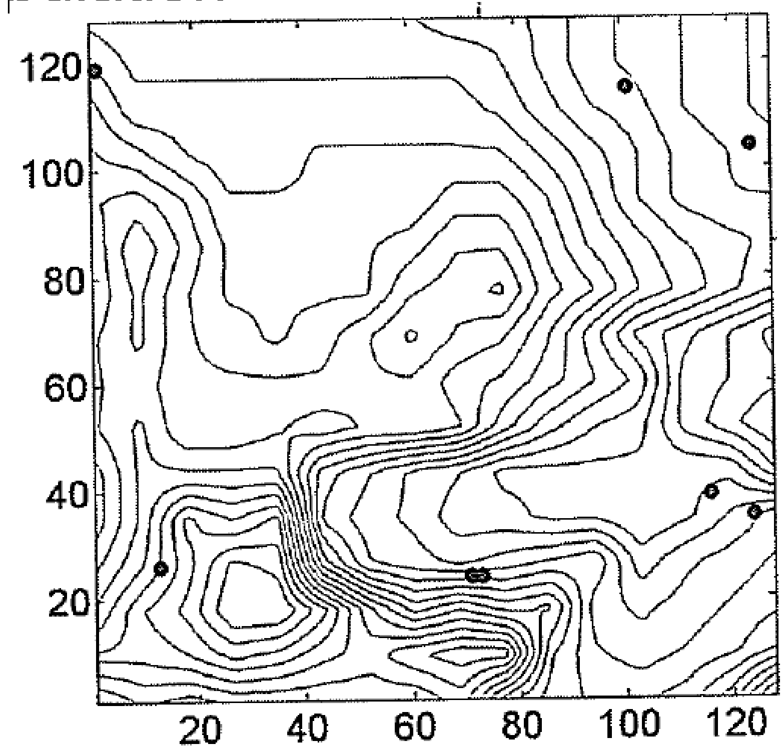
Adapted from [1].

Example 2D problem

Initial population

TABLE 2.3 Example Initial Population of 8 Random Chromosomes and Their Corresponding Cost

Chromosome	Cost
*00101111000110	−12359
11100101100100	−11872
*00110010001100	−13477
*00101111001000	−12363
11001111111011	−11631
01000101111011	−12097
*11101100000001	−12588
01001101110011	−11860



Adapted from [1].

* best chromosomes

Example 2D problem

Natural selection

Best 50%
Nkeep = 4

TABLE 2.4 Surviving Chromosomes after a 50% Selection Rate

	Chromosome	Cost
*	00110010001100	–13477
*	11101100000001	–12588
*	00101111001000	–12363
*	00101111000110	–12359

$$N_{keep} = X_{rate} N_{pop}$$

Adapted from [1].

Example 2D problem

Crossover

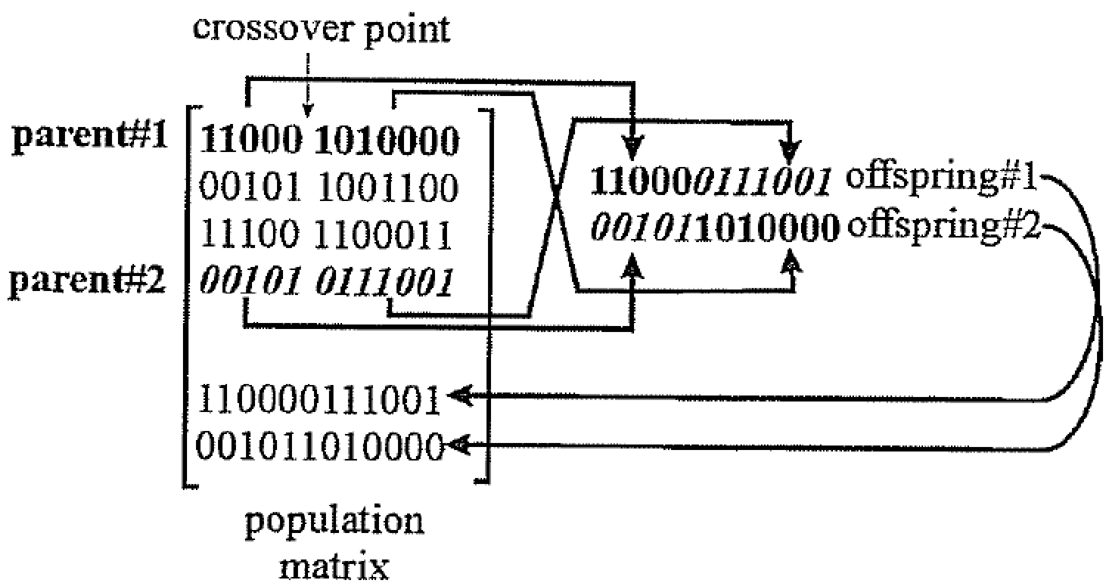


Figure 2.11 Two parents mate to produce two offspring. The offspring are placed into the population.

Adapted from [1].

Example 2D problem

Create offspring and replace bad chromosomes

TABLE 2.7 Pairing and Mating Process of Single-Point Crossover

Chromosome	Family	Binary String
3	ma(1)	*00101111001000*
2	pa(1)	11101100000001
5	*offspring*$_1$	*00101*100000001
6	*offspring*$_2$	11101*111001000*
3	ma(2)	*00101111001000*
4	pa(2)	00101111000110
7	*offspring*$_3$	*00101111000*110
8	*offspring*$_4$	00101111001*000*

Adapted from [1].

Example 2D problem

New population after mating

TABLE 2.8 Mutating the Population

Population after Mating	Population after Mutations	New Cost
00110010001100	00110010001100	–13477
11101100000001	11101100000001	–12588
00101111001000	00101111010000	–12415
00101111000110	00001011000111	–13482
00101100000001	00101000000001	–13171
11101111001000	11110111010010	–12146
00101111000110	00100111001000	–12716
00101111001000	00110111001000	–12103

Adapted from [1].

Example 2D problem

Members of population after first generation

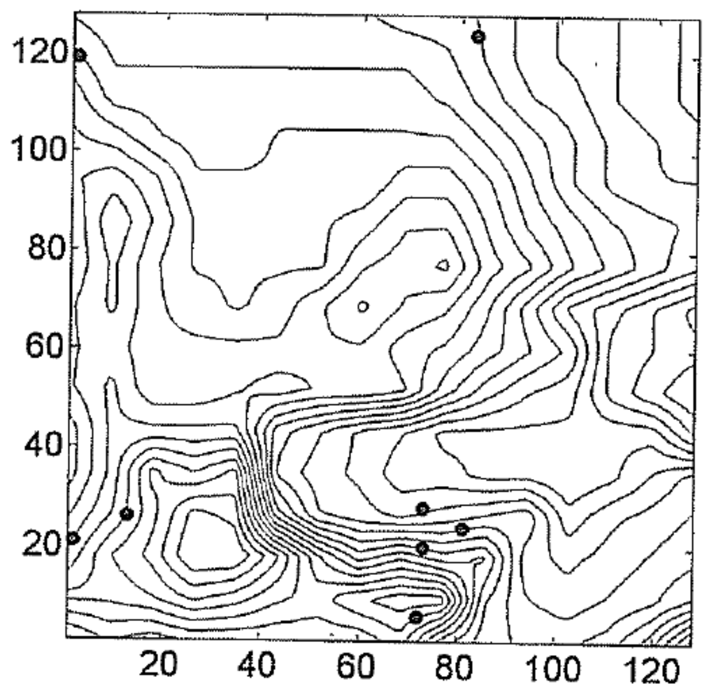


Figure 2.12 A contour map of the cost surface with the 8 members at the end of the first generation.

Adapted from [1].

Example 2D problem

New ranked population at start of second generation

New best chromosome →

TABLE 2.9 New Ranked Population at the Start of the Second Generation

Chromosome	Cost
00001011000111	–13482
00110010001100	–13477
00101000000001	–13171
00100111001000	–12716
11101100000001	–12588
00101111010000	–12415
11110111010010	–12146
00110111001000	–12103

Adapted from [1].

Example 2D problem

Population after crossover/mutation in 2nd generation

TABLE 2.10 Population after Crossover and Mutation in the Second Generation

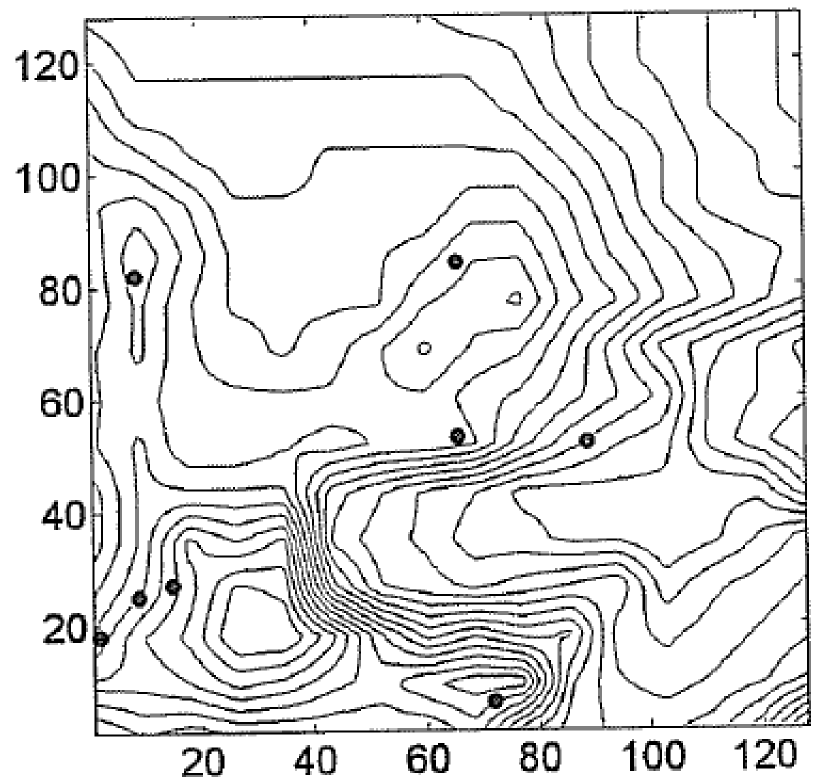
Chromosome	Cost
00001011000111	–13482
00110000001000	–13332
01101001000001	–12923
01100111011000	–12128
10100111000001	–12961
10100010001000	–13237
00110100001110	–13564
00100010000001	–13246

Note that 2nd best chromosome has been replaced by one with higher cost

Adapted from [1].

Example 2D problem

Members of population after 2nd generation



Adapted from [1].

Example 2D problem

- Example converged after only 3 gen's
- Height found: 14 199 m

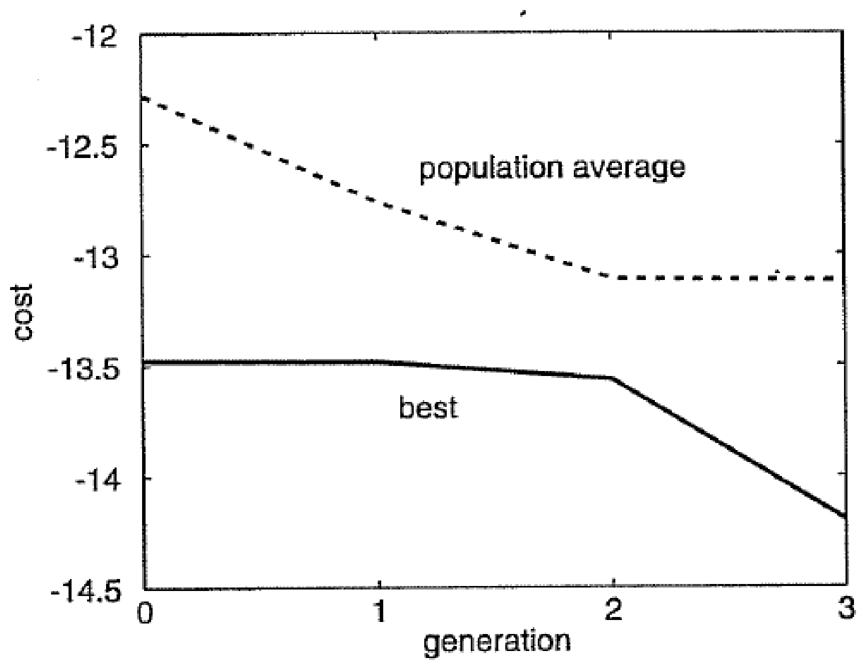


Figure 2.15 Graph of the mean cost and minimum cost for each generation.

Adapted from [1].

References

[1] Haupt & Haupt, Practical Genetic Algorithms, 2nd Ed., Wiley, 2004.