The existensial quantifier

Predicate logic

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Recall yesterday's example

1. $S(x) := x$ enters with a wet raincoat
2. $t :=$ it is raining outside
3. $s :=$ there is some $x \in C,$ such that $P(x)$

- $S(x)$ is a statement about some element x
- s is a statement about the universe C

Quantifiers

Definition

A quantifier is an expression or operator which turns a statement an arbitrary element into a statement about a universe.

- *there is some* is an existential quantifier
- A suitable element exists in the universe
- $\exists$ is the mathematical shorthand
  - $s = \exists x \in C,$ such that $P(x)$

$$s = \exists x \in C, P(x)$$

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Proof by example

$$s := \exists x \in C, P(x)$$

How do we prove s?

- We can identify some $x_0 \in C$ where $P(x_0)$ is true.
- Since $x_0 \in C \wedge P(x_0)$,
  - we can conclude $\exists x \in C, P(x)$

Principle (Proof by example)

To show that a statement with existential quantifier, it is sufficient to identify one value of the variable that gives a true statement.

Exercise

An equation can be thought of as a predicate. Consider the equation $x^2 + 1 = 0$.
Express the claim that the equation has a solution in symbolic form.