Formulæ and Definitions

A Reference for Discrete Mathematics

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Choosing k elements from an n-set

	Without replacement	With replacement
Ordered	k-element permutation $\frac{n!}{(n-k)!}$ possibilities	k-element list n^k possibilities
Unordered	subset $\binom{n}{k}$ possibilities	multiset (see Stein *et al*)

Partitioning

Definition (Partitioning)

A family of sets $\{S_1, S_2, \dots S_k\}$ is a partitioning of S if and only if

1. $S = \bigcup_{i=1}^{n-1} S_i$
2. $S_i \cap S_j = \emptyset$ whenever $i \neq j$.

Counting a partitioned set

Definition (Sum Principle)

If a finite set S has been partitioned into blocks, then the size of S is the sum of sizes of the blocks.

Definition (Product Principle)

If a finite set S has been partitioned into

$$S = S_1 \cup S_2 \cup \ldots \cup S_n$$

and every block has size $|S_i| = m$, then

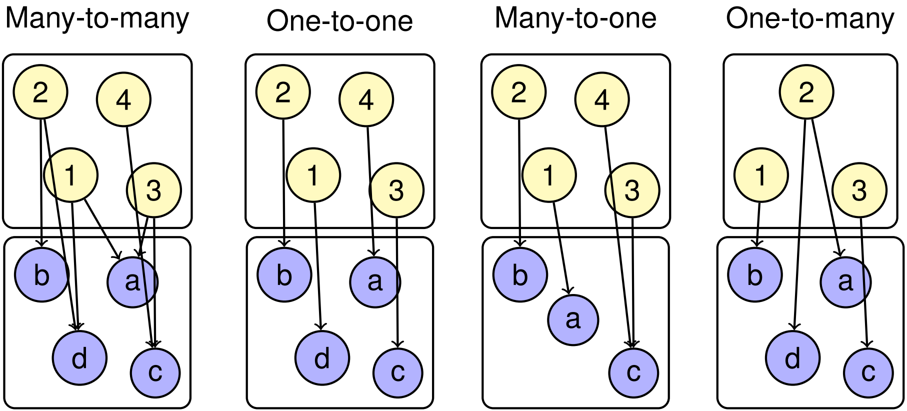
$$|S| = n \cdot m.$$

Relations

Definition

A relation from X to Y is a set R of ordered pairs (x, y) where $x \in X$ and $y \in Y$.

Different kinds of relations



Functions

Definition

A function $f : A \to B$ defines for value $x \in A$ a *function value* $f(x) \in B$.

Definition

Let $f : A \to B$ be a function. The domain of f is the set A. The co-domain of f is the set B.

Definition

Let $f : A \to B$ be a function. The range R of f is a subset of the domain defined as $R = \{f(x) | x \in A\}$.

Functions as Relations

- $R_f = \{(x, f(x)) : x \in X\}$
- Given x, there is a unique y, such that $(x, y) \in R_f$
- Given y, how many x exist such that $(x, y) \in R_f$?

General case could be 0, 1 or many

Surjective function every y is used
for any $y \in Y$, there is at least one x, such that $(x, y) \in R_f$

Injective function no y is used more than once
for any $y \in Y$, there is at most one pair $(x, y) \in R_f$

Definition

A Bijection is a function which is both injective and surjective

Equivalence Relations

Definition

A relation R on X is reflexive if xRx for any $x \in X$.

Definition

A relation R is symmetric if xRy whenever yRx.

Definition

A relation R is transitive if xRy and yRz implies that xRz.

Definition (Equivalence Relation)

A relation $\sim$ which is reflexive, symmetric, and transitive is called an equivalence relation.