The Binomial Coefficient

Subset counting and more

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Recollection

Remember the binomial coefficient

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

We defined this as the number of k-element subsets of an n-element set.

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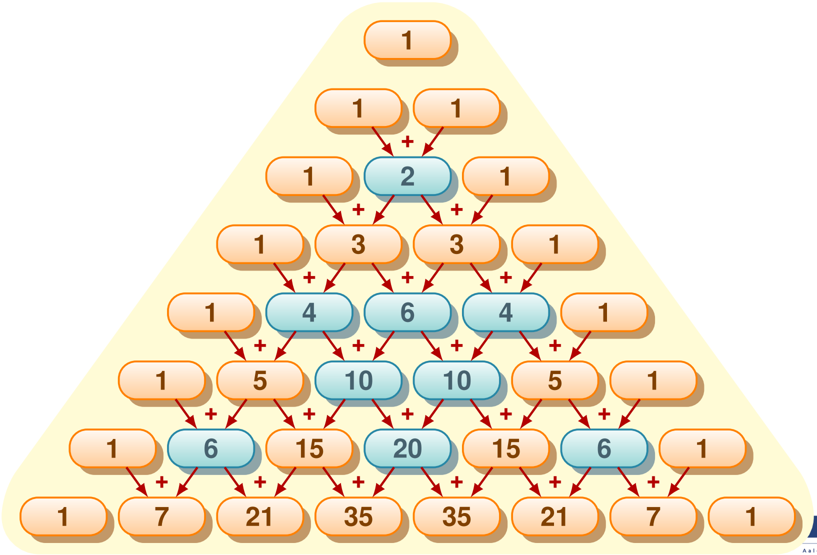
Pascal's Triangle



Pascal's Triangle

The formula

- Every cell is the sum of the two cells above it
  - except the 1-s at the borders
- This is the binomial coefficient $\binom{n}{k}$
  - Rows numbered $n = 0, 1, \ldots$
  - Entries numbered $k = 0, 1, \ldots, n$

The binomial coefficient

Recursive formula

$$\binom{n}{0} = 1 \quad \text{(unique empty set)}$$

$$\binom{n}{n} = 1 \quad \text{(unique complete set)}$$

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

Proof

Validity of the recursive formula

Why is $\binom{n}{k}$ the number of k-sets in an n-set?

- Aim: Choose a k-subset S from an n-set T.
- Consider an arbitrary element $x \in T$
  - 1-set $\{x\}$, and $(n-1)$-set $S \backslash \{x\}$
- Two alternatives for S
  1. either $x \in S$
  2. or $x \notin S$
- Case 1: we need a $(k-1)$-set from $T \backslash \{x\}$
- Case 2: we need a k-set from $T \backslash \{x\}$

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

Exercise

1. Calculate $\binom{12}{3}$ and $\binom{12}{9}$.
2. What can you say in general about $\binom{n}{k}$ in relation to $\binom{n}{n-k}$?