Functions as Relations

Surjective and Injective Functions

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Recollection

Relation between X and Y $R \subset X \times Y$

Function from X and Y $f : X \to Y$

The Function as a Relation

- $f : X \to Y$
- Defines a relation $R_f = \{(x, f(x)) : x \in X\}$
- Two special features
  - every $x \in X$ occurs in a pair $(x, y) \in R_f$
  - $x \in X$ cannot occur more than once in a pair $(x, y) \in R_f$
- A relation (in general) does not need either of these features

Injective and Surjective Functions

- $R_f = \{(x, f(x)) : x \in X\}$
- Given x, there is a unique y, such that $(x, y) \in R_f$
- Given y, how many x exist such that $(x, y) \in R_f$?

General case
- could be 0, 1 or many

Surjective function
- every y is used
- for any $y \in Y$, there is at least one x, such that $(x, y) \in R_f$

Injective function
- no y is used more than once
- for any $y \in Y$, there is at most one pair $(x, y) \in R_f$

Bijections

Definition

A Bijection is a function which is both injective and surjective

- A bijection is also called a one-to-one function
  1. Given y, there is a unique x such that $(x, y) \in R_f$
  2. Given x, there is a unique y such that $(x, y) \in R_f$
- There is an inverse function $f^{-1} : Y \to X$
  1. $f^{-1}(f(x)) = x$
  2. $f(f^{-1}(y)) = y$
- There corresponding relation is
  - $R_{f^{-1}} = \{(y, x) : (x, y) \in R_f\}$

Exercise

- Consider weekend activities
    - Set of activities $A = \{\text{Horseriding}, \text{Badminton}, \text{BBQ}\}$
    - Set of days $D = \{\text{Saturday}, \text{Sunday}\}$

1. List all possible functions $A \to D$
2. List all possible functions $D \to A$
3. Which of the functions are injective?
4. Which of the functions are surjective?