Unordered selection

Sets versus lists and permutations

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July 8, 2014

Different patterns of selection

	Without replacement	With replacement
Ordered	k-element permutation $\frac{n!}{(n-k)!}$	k-element list n^k
Unordered	subset $\binom{n}{k}$	multiset (see Stein *et al*)

Selection without replacement

k-element permutation: order matters

Set: order does not matter

1. Many different k-element permuations
   - correspond to one k-element subset

Counting

How many k-element subsets exist in an n-set?

- We know how to count k-element permutations
  - $\frac{n!}{(n-k)!}$
- but this counts each subset many times
  - once for every possible ordering of the elements

In how many ways can we order the elements of a k-set?

Quotient Principle

Count k-element permutations in two steps.

- Choose a k-element permutation from an n-set T
  1. choose a k-element subset $D \subset T$
  2. choose a permuation of D

Definition (Quotient Principle)

If we can partition a set of size p into q blocks of size r, then $q = p/r$.

The binomial coefficient

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

- Read: *n choose k*
- It is the number of ways to choose k out of n

Exercise

Exercise

A 20-person club are going to elect a board. In how many ways can the elect ...

1. *four members for the board?*
2. *a chair (1), a vice chair (2), a treasurer (3), and a secretary (4) to make up the board?*

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