## Functions

Lists generalised

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## Functions

$$
f: S \rightarrow T
$$

- Relates an element $f(x) \in T$ for every $x \in S$
- $S$ is called the domain
- $T$ is sometimes called the codomain
- The range $R_{f}$ of $f$ is
- set of values ever assumed by $f(x)$
- $R_{f}=\{f(x): x \in S\}$
- $R_{f} \subset T$ (subset of $T$ )


## A Programmer's View

Functions

Methods and functions in programming are in principle functions.

- Input arguments in $S\left(=S_{1} \times \ldots \times S_{n}\right)$
- Return value in $T$
- Data types are sets


## The list as a function

A list of $k$ elements from an n-set $T$.

$$
L=\left[x_{1}, x_{2}, \ldots, x_{k}\right]
$$

- Indexing gives us a map $i \mapsto x_{i}$
- i.e. function $L: \mathbb{N}_{n} \rightarrow T$
- $\mathbb{N}_{n}=\{1,2, \ldots, k\}$ is the natural numbers up to $k$ inclusive

Lists share key properties with the set of natural numbers.

## Counting possible functions

How many different functions $f: X \rightarrow Y$ exist from the $k$-set $X$ to the $n$-set $Y$ ?

| 1. | $x_{1}$ |  |
| :--- | :--- | :--- |
| 2. | $x_{2}$ |  |
| 3. | $x_{3}$ |  |
| 4. | $x_{4}$ |  |
| $\vdots$ |  |  |
| $k$ | $x_{k}$ |  |

- Write the elements of $X$ as a list
- arbitrary order
- Count as we did for a list
(1) You have $k$ slots to fill.
(2) Each slot gives you $n$ options.
(3) Use the Product Principle


## Exercise

You are going to hand out $k$ distinctly coloured balloons at a birthday party of $n$ children. In how many ways can the $k$ balloons be distributed to the $n$ children, with no limit on the number of balloons a single child may receive?

